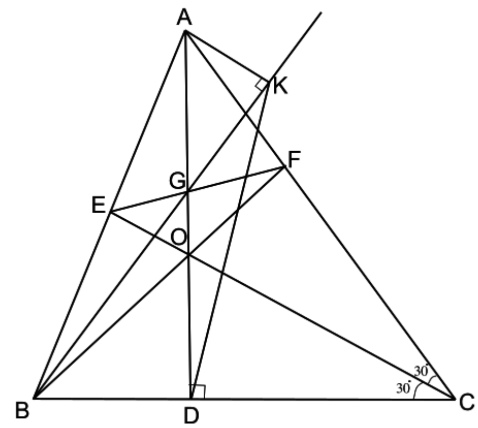
II Prize Winner Mr. Peeyush Pande's Solution

Given :



$\angle ACB = 60°$ and CE bisects $\angle ACB$

AD⊥BC, AK⊥ BK

To prove : $BK^2 = BD^2 + DK^2 + 2AK^2$

In ΔODC

$\angle OCD = 30°$ $\quad \angle ODC = 90°$

∴ It is 30° -60° -90° right Δ

∴ OD = $\frac{1}{2}OC$

$\angle DAC = 90° - 60° = 30°$

∴ $OA = OC$ $\quad (\angle OAC = \angle OCA = 30°$

$\Longrightarrow OD = \frac{1}{2}AO \quad \Longrightarrow AO = \frac{2}{3} AD$

According to concurrency theorem

$\frac{AG}{OG} = \frac{AD}{OD} \Longrightarrow \frac{AG}{OG} = \frac{3}{1}$

∴ AG = $\frac{3}{4} \times AO = \frac{3}{4} \times \frac{2}{3} AD = \frac{AD}{2}$

∴ G is midpoint of AD implies AG = GD = $\frac{AD}{2}$

∴ In Δ AKD , by Apollonius Theorem,

$AK^2 + DK^2 = 2(AG^2 + GK^2)$

AGK - Right Δ

$AG^2 = AK^2 + GK^2$

$AK^2 + DK^2 = 2AK^2 + 2GK^2 + 2GK^2$

$\Longrightarrow DK^2 = AK^2 + 4GK^2$

$\Longrightarrow DK^2 = AK^2 + 4AG^2 - 4AK^2$

$\Longrightarrow DK^2 + 3AK^2 = (2AG)^2$

$\Longrightarrow DK^2 + 3AK^2 = AD^2$

$\Longrightarrow DK^2 + 2AK^2 - AD^2 + AB^2 = -AK^2 + AB^2$

$\Longrightarrow DK^2 + 2AK^2 + BD^2 = BK^2$